A simple tool to understand patterns of electoral volatility

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Abstract

This paper shows how decentralized election results can be used to understand patterns of electoral volatility. Comparing two elections, the model provides a lower bound of the share of voters lost by a party, and of the number of new voters that it managed to convince. The paper proposes a typology of electoral results based on the estimated parameters of the model. We argue that the model can be used to better understand the local specificities linked to the personalization of politics, to avoid errors linked to omitting the role of abstention, and that it also provides a simple tool for better during the fact prediction of electoral results.

Keywords

Volatility; Electoral modelling; Personalization; Forecasting

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1. Introduction

Understanding electoral volatility matters. One the one hand, it is an essential part of the debate on party system stability (Dalton and Wattenberg, 2000), institutionalization (Mainwaring and Zoco, 2007), or party decline (Drummond, 2007). On the other hand, from a more practical viewpoint, it is a crucial element in the interpretation of electoral results. Electoral results are often interpreted at the aggregate level. From that perspective, electoral volatility corresponds to the aggregate turnover from one party to other parties, from one election to the next (Bartolini and Mair, 1990; Pedersen, 1979; Tavits, 2005). When conducting the analysis at the aggregate level, the risk is to take a binary view opposing winners and losers. In this way, it is often implicitly understood that the votes lost by a (losing) party simply benefit the winning party. This simplification is a typical example of ecological fallacy (Pilet, 2008). The analysis at the aggregate level hides some transfers between parties; furthermore it does not allow to distinguish between two very different patterns: a stable electoral score might hide major transfers of votes from one election to the other (loss of voters and gain of new voters), or it might just reflect stable and loyal voters from one election to the other. To overcome this ecological fallacy, the main solution has been to move to the individual level via electoral surveys, and to look at shifts in electoral behavior from one election to the other (Hawley and Sagarzazu, 2012).

What we propose is somewhat different: we introduce a simple model based on decentralized results to analyse electoral volatility. This rather simple tool applies to all types of electoral and party systems. It shows how decentralized election results can be used to understand the movements of volatile voters and it allows to propose a typology of electoral results based on simple patterns of volatility. It also allows to measure some local specificities, to understand the real impact of individual politicians in a context of personalization of politics (Karvonen, 2010), and to provide a simple tool for better during the fact prediction of electoral results (Campbell and Lewis-Beck, 2008).

The estimated model is based on the variance between the (decentralized) results. By ‘result’, we mean the score of a party in a given election, compared to its score in the previous one. The result of a party in a given zone is the combination of a national trend and local characteristics. The key idea of our specification is that a given national trend translates in different ways in different zones, because the local results in the previous election also differ. If the national trend is that a party has lost half of its electorate since the previous election, the loss will not necessarily be the same in zones where the party previously reached a high score and in zones where it was rather weak. To take this into account, we divide the national trend into two parts. The first one is what we call the unconditional change: it represents the share of the electorate (which may be zero) that did not vote for the considered party at the previous election, but that the party managed to attract this time around. This share is assumed to be identical throughout the entire country under study. The second part is called conditional change: it is the proportion of the votes obtained by the considered party at the previous election that it managed to retain this time around. The same proportion, in different zones, leads to different results depending on the local score in the previous election. Taken together, these parameters allow to estimate, on average, the percentage of voters who voted for the party in this election but did not in the previous one and the share of voters who left the party with respect to the previous election.
This is a lower bound on the degree of volatility in the sense that it only captures the information from the variance of the results between the different electoral zones, but not the variance within each zone. Therefore, the smaller the decentralized unit, the more we expect to capture volatility.

The error term in our estimation, obviously unexplained by the model, corresponds to what we call ‘local characteristics’: the part of the result that is not explained by the national trend. What we show in this paper is that the local characteristics we identify may be very different from what an analyst may find if he ignores one of the two parameters of the national trend. In particular, our method helps to understand which local politicians actually outperformed or underperformed compared to the national trend (and not the national result). Besides, this simple and ready-to-use model can help to understand how abstention evolves in some zones. It also helps to avoid mistakes in during the fact predictions (see Foucart, Gassner and van Haute, 2012). The ambition is not to try to explain the reason why volatile voters may change their vote. We just provide a simple toolbox to understand how they change.

In the next section, we present the theoretical assumptions of the model and the typology of electoral results that we propose based on the potential patterns of volatility. Section 3 discusses the contribution of the model on four aspects: the identification of patterns of electoral results, the role of personalization of politics and local effects, the role of abstention and challenger parties, and the potentialities in terms of prediction of electoral results. We conclude in section 4.

2. The model

2.1. Setup

Assume there are $n$ parties, for which electoral results are available in $m$ decentralized zones. For an election $t$, we call electoral result of party $i$ in zone $j$, denoted $y_{i,j}^t$, party $i$’s share of the total vote in zone $j$.\footnote{We therefore make the implicit assumption that the total number of voters is constant. In practice, the size of the electorate may of course vary and, even if it remains constant, the identity of some voters may change. This implies that if a voter disappears and is replaced by a voter choosing the same party, our model will identify this ‘constant vote’ as a loyal voter. This is another reason why our model only gives a lower bound on the degree of volatility.} The corresponding level of abstention is $A_{t,j}$, such that

$$\sum_{i=1:n} y_{i,j}^t + A_{t,j} = 1$$

The result of party $i$ in election $t$ is thus an $mx1$ matrix $Y_t^i$, each of the $m$ rows of which contains its result in one of the zones.

Consider that the result of a party in a given zone at a given election is made of three components respectively corresponding to: new, loyal and local voters. The first component (new voters) corresponds to the share of new voters attracted by the party in this election ($i$). By new voters, we mean voters who did not vote for the party at the previous elections as well as first-time voters. Loyal voters are those who vote for the party in election $t$ and who
already chose the same party at the previous election \((t-1)\). The third component \((local \ voters)\) corresponds to the local specificities that are not captured by the national trend for the party. Result \(y_{t,j}^{t}\) can thus be rewritten as the sum of those three components:

\[ y_{t,j}^{t} = \alpha^{t} + \beta^{t}y_{t-1,j}^{t} + \epsilon^{t} \]

Where

- \(\alpha^{t}\) corresponds to the unconditional gain of new voters (we expect \(\alpha^{t} \geq 0\))
- \(\beta^{t}y_{t,j}^{t}\) corresponds to loyal voters (we expect \(0 \leq \beta^{t} \leq 1\))
- \(\epsilon^{t}\) corresponds to local specificities

The assumption that the loss of votes is proportional to the stock \((\beta^{t} \leq 1)\), while the gain is unconditional \((\alpha^{t} \geq 0)\) is not crucial to our model. However, we believe it is natural to restrict our analysis to this setting since a party has more to lose in zones where it was already strong. Also, we do not have any particular reason to believe that a higher initial stock would help in convincing new voters. Still, the model is not restricted by the bounds expected on the parameters and particular cases could easily be identified.\(^2\)

While a simple OLS regression could capture the general trend for each party, the error terms are likely to be correlated.\(^3\) Indeed, if a party was to reach a particularly high score in zone \(j\), it may be that other parties have suffered from a particularly low score. Thus, an accurate estimate must take into account the existence of potential correlation among error terms. In practice, this corresponds to estimating a set of \(n\) Seemingly Unrelated Regressions (Zellner, 1962).\(^4\) Finally, the model also assumes that abstention (and/or the score of smaller parties for which no estimation is made) is high enough to avoid estimating a sum of voting shares higher than one in some of the zones. In practice, this implies that the sum of the estimated results is lower than 1, this is:

\[ \sum_{t=1:n} y_{t,j}^{t} \leq 1 \]

---

\(^2\) Out of the 11 parties studied in Foucart, Gassner and van Haute (2012), we found no value of \(\alpha^{t}\) significantly lower than 0 and no value of \(\beta^{t}\) significantly higher than 1.

\(^3\) One could avoid this problem in a two-party setup, at the cost of not taking into account the role of abstention.

\(^4\) The use of seemingly unrelated regressions for the analysis of multiparty elections was pioneered by Tomz et al. (2002). Recent applications of this method include Alemán and Kellam (2008).
2.2. A typology of electoral results

Let us call *loyal to party i* the voters who always vote for party *i*. Will be called *volatile* the voters who can (but do not necessarily) vote for different parties at each election. What we estimate corresponds to the behaviour of volatile voters. A reasonable assumption is that the estimated value of $\beta_i$ is smaller than or equal to 1 for all $i$, as there is *no reason for the gain of new voters to depend on the previous score*. Indeed, the dynamics of the movements we have in mind is that, at each period, a proportion $\beta_i$ of the voters who had chosen party $i$ in $t-1$ vote for that party again in $t$. Those voters are both voters that are loyal to party $i$ and ‘volatile voters who decide to vote for the same party again’. Thus, when a party loses voters, the loss is considered to be proportional to the size of its electorate in $t-1$.

This contrasts with the gain of new voters. Some share of the electorate may be seduced by the party. These are volatile voters who did not vote for party $i$ in period $t-1$. There is no reason for this share to be proportional to the share of voters in a specific zone. Thus, for each party, our model estimates a lower bound (to repeat, the larger the electoral zone, the more of the information from the variance in volatility within each zone is missing) for:

- $\alpha_i$: The unconditional gain of new voters
- $1 - \beta_i$: The proportion of the previous voters who left the party

These estimates are lower bounds of the degree of volatility because they only capture the information from the differences between electoral zones, but not within each electoral zone. Thus, the higher the level of decentralization of the data, the higher the information captured by the model.

This allows us to classify the possible electoral results of party $i$ at one election with respect to the previous one into four categories on the basis of the values of $\alpha_i$ and $\beta_i$: the considered party may face a gain, renewal, stability, or a loss (Table 1).

Table 1. A typology of electoral results

<table>
<thead>
<tr>
<th>$\alpha_i$</th>
<th>$\beta_i$</th>
<th>Electoral result</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;0</td>
<td>1</td>
<td><strong>Gain</strong>: The party gains new voters and does not lose any</td>
</tr>
<tr>
<td>&gt;0</td>
<td>&lt;1</td>
<td><strong>Renewal</strong>: The party gains new voters but loses a share of its electorate</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td><strong>Stability</strong>: The party does not gain new voters but retains its electorate</td>
</tr>
<tr>
<td>0</td>
<td>&lt;1</td>
<td><strong>Loss</strong>: The party does not gain new voters and loses a share of its electorate</td>
</tr>
</tbody>
</table>

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We provide a formalization of this argument in appendix A.
3. Potential contributions of the model

3.1. Distinguishing patterns of electoral volatility

A renewed party is obviously quite different from a stable one. However, aggregate results do not allow to distinguish between the two patterns. Table 2 presents the results of two parties A and B in three different zones. For the moment, we assume that there is no abstention, and that we are not interested in the results of parties other than A and B. Party A is a stable party \((\alpha^A = 0\%, \beta^A = 1)\),\(^6\) while party B is a renewed party \((\alpha^B = +10\%, \beta^B = 0.6)\).\(^7\)

On average, in the three considered zones, the results of both parties are constant (same score at the two elections). However, the underlying process is different. For each party and each zone, we present two values: the difference \(\Delta\) between its scores at the two elections, and local specificities \(\varepsilon\) as estimated in the model. For party A, the two are identical. For party B, the values are very different. Ignoring this difference may lead to false interpretations. Our model allows to distinguish between the two patterns.

**Table 2. Renewal and stability in three zones**

<table>
<thead>
<tr>
<th>Zone</th>
<th>(y^A_{t-1})</th>
<th>(y^A_t)</th>
<th>(\Delta^A_t)</th>
<th>(\varepsilon^A)</th>
<th>(y^B_{t-1})</th>
<th>(y^B_t)</th>
<th>(\Delta^B_t)</th>
<th>(\varepsilon^B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>30%</td>
<td>33%</td>
<td>+3%</td>
<td>+3%</td>
<td>20%</td>
<td>20%</td>
<td>0%</td>
<td>-2%</td>
</tr>
<tr>
<td>Centre</td>
<td>20%</td>
<td>18%</td>
<td>-2%</td>
<td>-2%</td>
<td>40%</td>
<td>36%</td>
<td>-4%</td>
<td>+2%</td>
</tr>
<tr>
<td>South</td>
<td>10%</td>
<td>9%</td>
<td>-1%</td>
<td>-1%</td>
<td>15%</td>
<td>19%</td>
<td>+4%</td>
<td>+0%</td>
</tr>
<tr>
<td>Average</td>
<td>20%</td>
<td>20%</td>
<td>0%</td>
<td>0%</td>
<td>25%</td>
<td>25%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

3.2. Capturing local specificities

Consider an analyst asked to rank the performances of party B in the three zones, to find out which of the local politicians performed the best (for instance, to analyse the impact of the presence of a given personality on the results of an election). Table 3 presents the rankings, respectively calling on the observed differences \(\Delta\), and the true local specificities \(\varepsilon\).

**Table 3. Local specificities**

<table>
<thead>
<tr>
<th>Ranking of the zone</th>
<th>Method (\Delta)</th>
<th>Method (\varepsilon)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>South</td>
<td>Centre</td>
</tr>
<tr>
<td>2</td>
<td>North</td>
<td>South</td>
</tr>
<tr>
<td>3</td>
<td>Centre</td>
<td>North</td>
</tr>
</tbody>
</table>

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\(^6\) Party A keeps all its voters and does not attract new volatile voters.

\(^7\) Party B attracts 10\% of the electorate as ‘new volatile voters’ but loses 40\% of its previous voters.
An analyst able to correctly estimate $\varepsilon$ would understand that the ‘Centre’ region is actually the one in which party B has performed the best compared to its national trend, while an analyst erroneously focusing on differences $\Delta$ only would consider that the ‘Centre’ region is where party B obtained its worst local result. Our model allows to better capture local specificities, which is crucial in a context of the personalization of politics.

3.3. Taking into account the role of abstention and smaller parties

Introducing now the possibility of abstention as well as the existence of parties other than A and B, consider the results of parties A and B in zone ‘East’, with the same parameters as before ($\alpha^A = 0\%, \beta^A = 1, \alpha^B = +10\%, \beta^B = 0.6$). Together, the scores of all other parties amount to 30% of the votes, both in $t-1$ and $t$ (to simplify, we assume they do not attract volatile voters but rely on loyal voters only).

The key issue here is to understand the difference between the share $y$ of the electorate and the share $z$ of the votes cast\(^8\). Here, even if their results are very different, both A and B can be credited with the same good performance in the zone, performing better ($\varepsilon = +3\%$) than in the rest of the country (Table 4). Without taking into account the role of abstention, this ‘better’ performance would have been captured by the difference in terms of votes $\Delta y$. The problem is that, even when ignoring the good local performance, both parties see their score increase, therefore decreasing abstention. Thus, as abstention decreases, the share of votes actually cast increases, and a same number of voters represents a lower share of the cast votes $z$.

Table 4. The role of abstention

<table>
<thead>
<tr>
<th></th>
<th>$y_{t-1}$</th>
<th>$z_{t-1}$</th>
<th>$y_t$</th>
<th>$z_t$</th>
<th>$\Delta y$</th>
<th>$\Delta z$</th>
<th>$\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Party A</td>
<td>40%</td>
<td>50%</td>
<td>43%</td>
<td>46.7%</td>
<td>+3%</td>
<td>-3.3%</td>
<td>+3%</td>
</tr>
<tr>
<td>Party B</td>
<td>10%</td>
<td>12.5%</td>
<td>19%</td>
<td>20.7%</td>
<td>+9%</td>
<td>+8.2%</td>
<td>+3%</td>
</tr>
<tr>
<td>Others</td>
<td>30%</td>
<td>33.3%</td>
<td>30%</td>
<td>32.6%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Abstention</td>
<td>20%</td>
<td></td>
<td></td>
<td>8%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comparing the values of $z$ for parties A and B implies committing two errors. The first error consists in ignoring the trend and the role of $\varepsilon$. The second error is to ignore the fact that party B also did better in the zone. Therefore party A, who actually outperformed in the zone, is considered as having done worse than in $t-1$ ($\Delta z$ is strictly negative). Our model allows to avoid these two errors.

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\(^8\) i.e. disregarding abstention.
3.4. Carrying out *During the fact* predictions

Consider now an analyst aiming at predicting the electoral results on the night of an election (*during the fact* predictions, Campbell and Lewis-Beck, 2008), based on preliminary results gathered in decentralized zones (Table 5). Suppose there are four parties, A, B, C, and D and, to simplify, no abstention.

**Table 5. True parameters for four types of patterns**

<table>
<thead>
<tr>
<th>Pattern</th>
<th>$Y_{t-1}$</th>
<th>$Y_t$</th>
<th>$\beta$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: renewal</td>
<td>20%</td>
<td>20%</td>
<td>0.5</td>
<td>+10%</td>
</tr>
<tr>
<td>B: loss</td>
<td>30%</td>
<td>15%</td>
<td>0.5</td>
<td>0%</td>
</tr>
<tr>
<td>C: gain</td>
<td>15%</td>
<td>30%</td>
<td>1</td>
<td>+15%</td>
</tr>
<tr>
<td>D: stability</td>
<td>35%</td>
<td>35%</td>
<td>1</td>
<td>0%</td>
</tr>
</tbody>
</table>

Let $Y_{t-1}$ and $Y_t$ represent respectively average national scores in $t-1$ and $t$. Consider two types of analysts, basing their predictions on different intuitions. The ‘unconditional’ forecaster predicts a global result, by simply considering that observed local differences $\Delta$ represent a national trend, i.e. that global scores are simply increased by $\Delta$. The ‘linear’ forecaster, on the other hand, wants to take into account local differences, and considers that the proportional change in local zones should represent the national trend, i.e. that global scores are simply multiplied by a factor that is constant across zones.

We assume all analysts have enough decentralized data to correctly estimate the parameters and, for simplicity, we assume there are no local specificities in the volatility ($\varepsilon=0$). The information contained in the available results allows us to correctly estimate the parameters, and therefore identify the national trend. However, they are collected in zones where the number of loyal voters of each party largely differs from the national trend. This difference is what would lead both the unconditional and the linear forecaster to make errors in their predictions. Denote the partial results obtained by $Y^P_t$ and the results from the same zones in the previous election by $Y^P_{t-1}$. The predicted national scores $\hat{Y}_t$ of the two analysts are given in Table 6.

**Table 6. During the fact predictions**

<table>
<thead>
<tr>
<th>Pattern</th>
<th>$Y_{t-1}$</th>
<th>$Y_t$</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>$Y^P_{t-1}$</th>
<th>$Y^P_t$</th>
<th>$\hat{Y}_t$ unconditional</th>
<th>$\hat{Y}_t$ linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: renewal</td>
<td>20%</td>
<td>20%</td>
<td>0.5</td>
<td>+10%</td>
<td>40%</td>
<td>30%</td>
<td>10%</td>
<td>15%</td>
</tr>
<tr>
<td>B: loss</td>
<td>30%</td>
<td>15%</td>
<td>0.5</td>
<td>0%</td>
<td>10%</td>
<td>5%</td>
<td>25%</td>
<td>15%</td>
</tr>
<tr>
<td>C: gain</td>
<td>15%</td>
<td>30%</td>
<td>1</td>
<td>+15%</td>
<td>35%</td>
<td>50%</td>
<td>30%</td>
<td>21.43%</td>
</tr>
<tr>
<td>D: stability</td>
<td>35%</td>
<td>35%</td>
<td>1</td>
<td>0%</td>
<td>15%</td>
<td>15%</td>
<td>35%</td>
<td>35%</td>
</tr>
</tbody>
</table>
The unconditional analyst correctly predicts the results of party C (gain) and party D (stability). The linear analyst correctly predicts the results of party B (loss) and party D (stability). None of them manages to correctly predict the results of party A. In theory, an analyst using our model and estimates would be able to correctly predict all results. Understanding the cases in which each type of forecaster is the more likely to be wrong is crucial when one wants to use forecasting models in multipartisan settings (Dassonneville and Hooghe, 2012). The general intuition we want to emphasize is that, when observing the first electoral trends, an analyst must at least treat the increase in votes of a party and the share of the voters that it loses in a different way.

4. Conclusion

This paper focuses on the phenomenon of electoral volatility from a new angle. It takes a middle-ground approach between analyses conducted at the party system level, that look at volatility at the aggregate level, and studies of electoral behaviour, that look at individual shifts between parties, translating volatility as a transfer of vote. Our middle-ground approach looks at disaggregated electoral results, preferably at the most decentralized level. It shows how these results can be used in a simple model to understand patterns of volatility.

The model uses the variance contained between the decentralized results, and provides a lower bound of the proportion of voters lost by a party from one election to the next, and of the share of new voters that have been convinced to vote for it. In this model, the smaller the decentralized unit, the more it captures volatility.

On the basis of the model, we set up a typology of electoral results based on four potential patterns of volatility. Error terms in the model also allow to better capture local specificities, thereby taking into account the effects of personalization of politics. Furthermore, the model helps to avoid interpretation errors linked to the omission of abstention. Finally, it could also serve as a simple tool for better during the fact prediction of electoral results.

The simplicity of the model makes it applicable to all types of electoral formulas and party system types; it is a ready-to-use toolbox to understand patterns of volatility in various settings.

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9 To make the most of the information contained in the variance, we deliberately assume that all zones represent the same number of voters. The drawback of this assumption is that the estimator may be biased due to the fact that too much weight is awarded to the information from the smallest zones.
Acknowledgment

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Appendix A: A formalization of patterns of volatility

The result of party i in zone j in election t is given by:

$$y_{t,j}^i = p_j + \alpha_t + k_t (\alpha_{t-1} + \varepsilon_{t-1,j}) + \varepsilon_{t,j}$$  (1)

Where P represents the loyal voters, and k the share of volatile voters from the previous period the party managed to keep. For the ease of the proof, consider the polar cases of k=0 and k=1.

In t-1, (1) rewrites:

$$y_{t-1,j}^i = p_j + \alpha_{t-1} + k_{t-1} (\alpha_{t-2} + \varepsilon_{t-2,j}) + \varepsilon_{t-1,j}$$  (2)

The right-hand side of (2) can be decomposed in two parts: a constant Pj and a time-dependent share $\Psi_{t-1}$.

$$\alpha_{t-1} + k_{t-1} (\alpha_{t-2} + \varepsilon_{t-2,j}) + \varepsilon_{t-1,j} = \Psi_{t-1} y_{t-1,j}^i$$  (3)

This share $\Psi_{t-1}$ can itself be decomposed in a part $\lambda_{t-1}$ coming from election t-1 and a part $(1-\lambda_{t-1})$ coming from volatile voters obtained in t-2 and potentially conserved. Thus:

$$\alpha_{t-1} + \varepsilon_{t-1,j} = \lambda_{t-1} \Psi_{t-1} y_{t-1,j}^i$$  (4)

$$k_{t-1} (\alpha_{t-2} + \varepsilon_{t-2,j}) = (1 - \lambda_{t-1}) \Psi_{t-1} y_{t-1,j}^i$$  (5)

Isolating $p_j$ in (2) and putting it in (1), we get:

$$y_{t,j}^i = y_{t-1,j}^i - [\alpha_{t-1} + k_{t-1} (\alpha_{t-2} + \varepsilon_{t-2,j}) + \varepsilon_{t-1,j}] + \alpha_t + k_t (\alpha_{t-1} + \varepsilon_{t-1,j}) + \varepsilon_{t,j}$$  (6)

Using (4) and (5), equation (6) rewrites:

$$y_{t,j}^i = (1 - \Psi_{t-1} + k_t \lambda_{t-1} \Psi_{t-1}) y_{t-1,j}^i + \alpha_t + \varepsilon_{t,j}$$  (7)

Denote by $\beta_t = 1 - \Psi_{t-1} + k_t \lambda_{t-1} \Psi_{t-1}$, our estimate of equation (7) is thus:

$$y_{t,j}^i = \beta_t y_{t-1,j}^i + \alpha_t + \varepsilon_{t,j}$$

So, depending on the values of $kt, kt-1$, the potential values of $\beta_t$ are:

<table>
<thead>
<tr>
<th>$k_{t-1}$</th>
<th>$k_t=0$</th>
<th>$k_t=1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{t-1} - \Psi_{t-1}$</td>
<td>$\beta_{t-1}$</td>
<td>$\beta_{t-1} + \lambda_{t-1} \Psi_{t-1}$</td>
</tr>
<tr>
<td>$\beta_{t-1} - \Psi_{t-1}$</td>
<td>$\beta_{t-1}$</td>
<td>$\beta_{t-1} + \lambda_{t-1} \Psi_{t-1}$</td>
</tr>
</tbody>
</table>

As $\lambda_{t-1} \leq 1$, $\beta_t$ is, in any case, lower or equal to 1.
References


